Mr. Michael T. Davis WLPCS Calculus	Sections 5.2 & 5.3 Quiz April 13, 2018

Name: Solutions / Answers

Directions: Choose 2 of the 3 questions to answer. Show all your work carefully. 40 total points.

1. Given the function $f(x)=10x-x^2$, use the first derivative and the second derivative to determine:

a. (6 points) The interval(s) on which the function is increasing. Justify your answer.

$f'(x)=10-2x \quad 0=10-2x \quad x=5$ c.p.

$+ + + + \quad 5 \quad - - -$ f'

$f(x)$ is increasing on $(-\infty,5]$ since $f'>0$

b. (2 points) The interval(s) on which the function is decreasing. Justify your answer.

referring to the sign analysis in part a. above, $f(x)$ is decreasing on $[5,\infty)$ since $f'<0$

c. (3 points) The x value for which the graph of $f(x)$ has a point of inflection. Justify your answer.

$f''(x)=-2$ $\quad - - - - - - - -$ f''

$f(x)$ has no point of inflection since $f''(x)$ never changes sign

d. (2 points) The interval(s) on which the graph of $f(x)$ is concave up. Justify your answer.

$f(x)$ is concave up never since f'' is never positive.

e. (2 points) The interval(s) on which the graph of $f(x)$ is concave down. Justify your answer.

$f(x)$ is concave down on $(-\infty,\infty)$ since $f''<0$ on $(-\infty,\infty)$

f. (3 points) Use the second derivative test to identify any minimum or maximum values. Justify your answer.

$f(5) = 10(5) - 5^2 = 50 - 25 = 25$

$f(x)$ has an abs. max. of $y = 25$ at $x = 5$ since $f'(5) = 0$ and $f''(5) < 0$ in which case the graph of $f(x)$ is concave down at $x = 5$

2. Given the function $f(x) = xe^x + e^x$, use the first derivative and the second derivative to determine (If answered as an E.C., then count half points for each part).

a. (6 points) The interval(s) on which the function is increasing. Justify your answer. Justify your answer.

$f'(x) = 1 \cdot e^x + x \cdot e^x + e^x = 2e^x + xe^x$

$0 = e^x(2+x) \qquad x = -2 \text{ C.P.} \qquad e^x \neq 0$

$- - - \qquad + + + \quad f'$

-2

$f(x)$ is increasing on $[-2, \infty)$ since $f' > 0$

b. (2 points) The interval(s) on which the function is decreasing. Justify your answer.

$f(x)$ is decreasing on $(-\infty, -2]$ since $f' < 0$

c. (3 points) The x value of any point of inflection. Justify your answer.

$f''(x) = 2e^x + e^x + xe^x = 3e^x + xe^x$

$0 = 3e^x + xe^x \qquad 0 = e^x(3 + x) \qquad x = -3$

$e^x \neq 0$

$- - - \qquad + + + \quad f''$

-3

$f(x)$ has a P. of I. at $x = -3$ since $f''(-3) = 0$ and f'' changes sign at $x = -3$

d. (2 points) The interval(s) on which the graph is concave up. Justify your answer.

$f(x)$ is concave up on $[-3, \infty)$ since $f'' > 0$

e. (2 points) The interval(s) on which the graph is concave down. Justify your answer.

$f(x)$ is concave down on $(-\infty, -3]$ since $f'' < 0$

f. (6 points) Use the second derivative test to identify any minimum or maximum values. Justify your answer.

$f(-2) = -2e^{-2} + e^{-2} = -e^{-2} = \frac{-1}{e^2}$

$f(x)$ has a relative min. of $y = \frac{-1}{e^2}$ at $x = -2$ since $f'(-2) = 0$ and $f''(-2) > 0$ in which case the graph of $f(x)$ is concave up at $x = -2$.

3. Given the function $f(x)=\frac{x^3}{3}+2x^2-12x-5$, use the first derivative and the second derivative to determine:

a. (6 points) The interval(s) on which the function is increasing. Justify your answer.

$f'(x) = x^2+4x-12 \quad 0=(x+6)(x-2)$

$x=-6$ c.p. $\quad x=2$ c.p.

$+++ \quad --- \quad +++ \quad f'$

$-6 \quad 2$

$f(x)$ is increasing on $(-\infty,-6)\cup(2,\infty)$ since $f'(x)>0$

b. (2 points) The interval(s) on which the function is decreasing. Justify your answer.

$f(x)$ is decreasing on $(-6,2)$ since $f'(x)<0$

c. (4 points) The x value of any point of inflection. Justify your answer.

$f''(x) = 2x+4 \quad 0=2x+4 \quad x=-2$

$---- \quad +++ \quad f''$

-2

$f(x)$ has a point of inflection at $x=-2$ since $f''(-2)=0$ and f'' changes sign at $x=-2$

d. (2 points) The interval(s) on which the graph is concave up. Justify your answer.

$f(x)$ is concave up on $(-2, \infty)$ since $f' > 0$

e. (2 points) The interval(s) on which the graph is concave down. Justify your answer.

$f(x)$ is concave down on $(-\infty, -2)$ since $f' < 0$

f. (6 points) Use the second derivative test to identify any minimum or maximum values. Justify your answer.

$f(-6) = -72 + 72 + 72 - 5 = 67$

$f(2) = \frac{8}{3} + 8 - 24 - 5 = -18\frac{1}{3}$

$f(x)$ has a rel. max. of $y = 67$ at $x = -6$ since $f'(-6) = 0$ and $f''(-6) < 0$

$f(x)$ has a rel. min. of $y = -18\frac{1}{3}$ at $x = 2$ since $f'(2) = 0$ and $f''(2) > 0$